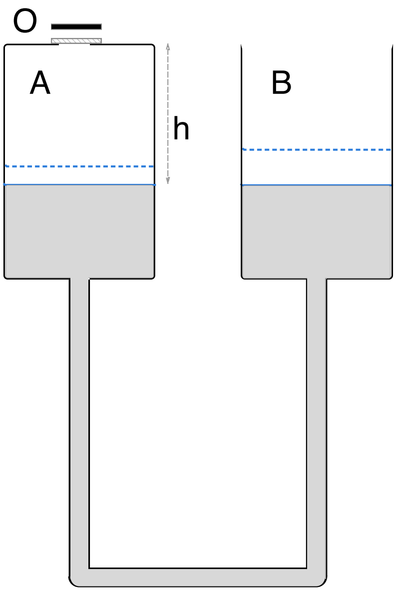
Problem 1



Two tanks of equal cross section of 100 cm^2 are connected by a tube as shown. Initially A is open to the atmosphere and water in both tanks is at the same level. The hole O is then closed and water is added in tank B until the level in tank A rises by $\frac{h}{20}$.

1- Assuming air is an ideal gas and that the temperature does not change:

(a) What will be the new pressure in tank A

(b) Knowing that the amount of water added is 7 liters, what is the volume of air contained in tank A before closing the tap and adding this volume of water to tank B.

2- After closing the tap and adding 7 liters to tank B, the temperature of A increases by 40°. Will the level in A go up, down, or remain unchanged?

Solution

Q .1

Consider tank A and let 1 and 2 refer to conditions before and after closing the tap. Assuming air to be ideal gas, we can write

$$p_1 V_1 = p_2 V_2; \; p_2 = \frac{p_1 V_1}{V_2} = p_1 \frac{20}{19} = 1.052 \, p_{atm} = 106.32 \text{ kPa}$$

$$\frac{20.}{19} 101$$

106.316

Q .2

After closing the tap and adding water to tank B, the level in B must be above that of the level in A by a quantity z, such as

$\gamma_w\, z = 6.32\ \text{kPa}$

hence $z = \frac{6.32}{9.8} = 0.64\ \text{m}$

and hence

$Q = \left(0.64 + 2\,\frac{h}{20}\right)\frac{100}{100^2}$

$\left(0.64 + \frac{h}{10}\right)\frac{1}{100} = \frac{7}{1000}$

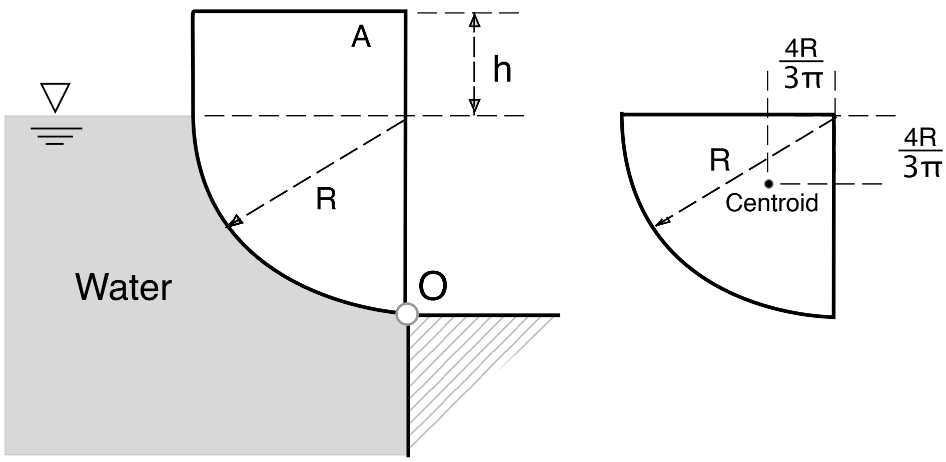
this gives $h = 0.6\ m$, hence the initial volume of air in tank A is $0.6\left(\frac{100}{100^2}\right) = 0.006\ m^3$

Q .3

If the temperature in A is $T_2 > T_1$ then $p_2 > p_1$ so thet the level in A will go down.

If the temperature in A is $T_2 < T_1$ then $p_2 < p_1$ so thet the level in A will go up.

Problem 2

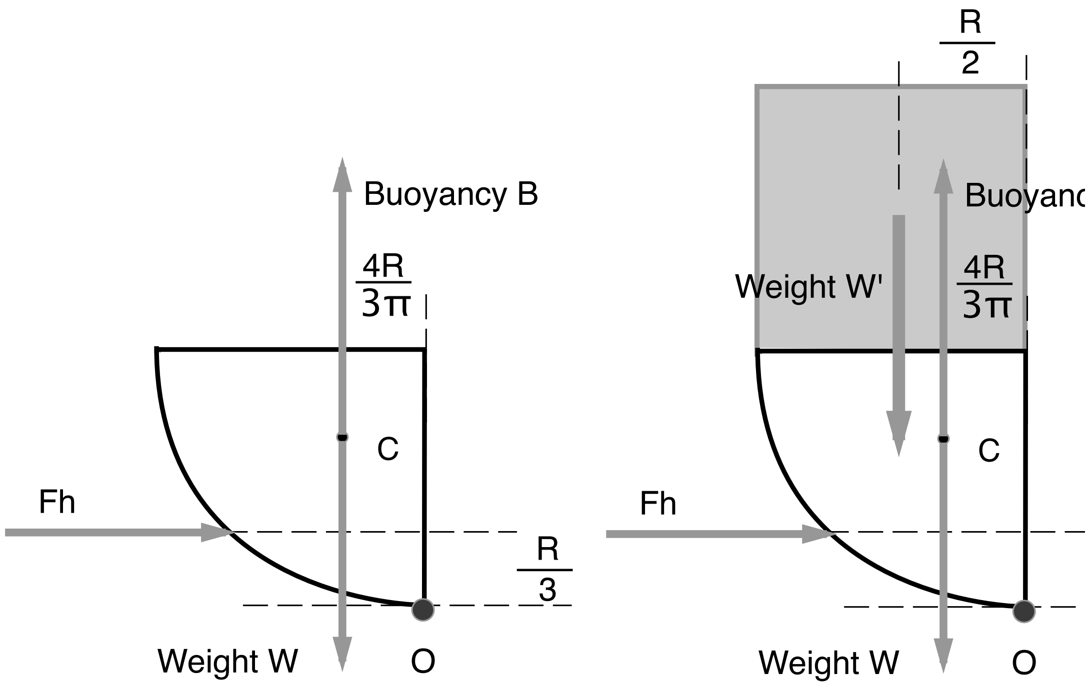


The uniform body A shown in the figure below has width b normal to the plane of the figure, and is in static equilibrium when pivoted about hinge O.The adjacent water is exerting a force on the body that is maintaining it in equilibrium.What is the density of this body if :

1- the distance h = 0

2- the distance h = R

Q.I



Free body diagram left figure with h = 0, all forces are shown except for the reaction at O since it produces no moment.

The weight of the body is

$$W = \gamma\,\text{Volume} = \gamma \frac{b\,\pi\,R^2}{4}$$

Buoyancy B (or vertical force),
is the weight of the volume of the quarter circle times its thikness with $\gamma = \gamma_{\text{water}}$

$$B = b\,\text{Volume} = \gamma_w \frac{b\,\pi\,R^2}{4}$$

The horizontal force is the same as if we had a rectangular vertical gate with height R and width b, hence

$$F_h = p_C\, b\,R = \gamma_w \frac{R}{2}\, b\,R$$

The vertical forces act on the centroid of the quarter cylinder as shown in the figure, while the horizontal force acts on a line $\frac{R}{3}$ above O, hence the moments of vertical forces around O

$$(B - W)\frac{4\,R}{3\,\pi} + F_h \frac{R}{3} = 0$$

$$\left(\gamma_w \frac{b\,\pi\,R^2}{4} - \gamma \frac{b\,\pi\,R^2}{4}\right)\frac{4\,R}{3\,\pi} + \gamma_w\, b\,\frac{R^2}{2}\,\frac{R}{3} = 0$$

$$b\,\frac{R^3}{3}\,(\gamma_w - \gamma) + \gamma_w\, b\,\frac{R^2}{2}\,\frac{R}{3} = 0$$

$$(\gamma_w - \gamma) + \gamma_w \frac{1}{2} = 0$$

$$\gamma = 3 \frac{\gamma_w}{2}$$

Q .2

Free body diagram right figure with h = R, all forces are shown except for the reaction at O since it produces no moment. In this case we have to add to the equation the moment of the weight of the upper part

$$W_2 = \gamma\, b\, R^2$$

Which acts at *a* distance $\frac{R}{2}$ from *O*, hence the equation of moment about *O* is :

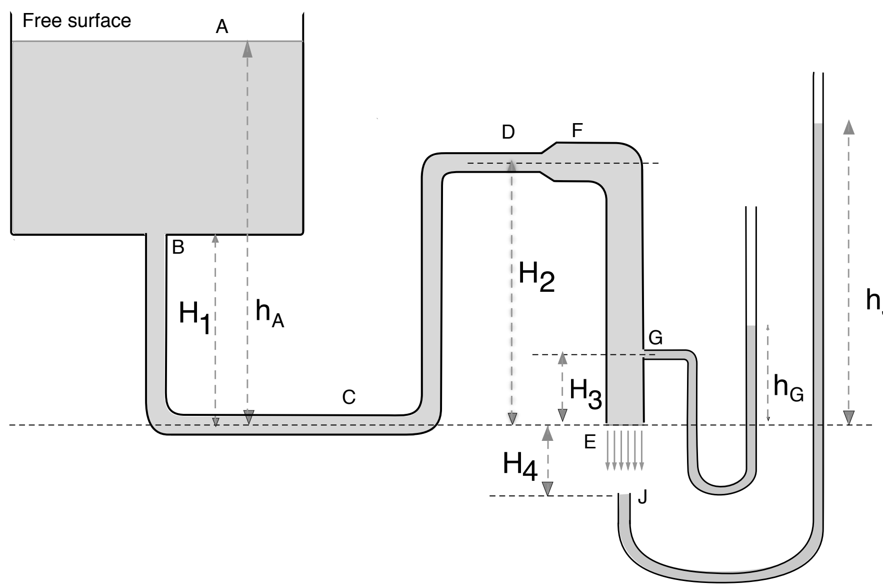
$$\left(\gamma_w \frac{b\,\pi\,R^2}{4} - \gamma \frac{b\,\pi\,R^2}{4}\right) \frac{4\,R}{3\,\pi} + \gamma_w\, b \frac{R^2}{2} \frac{R}{3} - \gamma\, b\, R^2 \frac{R}{2} = 0$$

$$\frac{b\,R^3}{3} (\gamma_w - \gamma) + \gamma_w\, b \frac{R^3}{6} - \gamma\, b \frac{R^3}{2} = 0$$

$$2\,(\gamma_w - \gamma) + \gamma_w - 3\,\gamma = 0$$

$$\gamma = \frac{3}{5} \gamma_w$$

Problem 3



Consider the flow from a large tank in the pipe system BCDFGE as shown. Note that the drawing is not to scale. The fluid is assumed to be incompressible and non-viscous. A liquid manometer is attached to the pipe at G, and a separate tube on the right is bent and inserted in the flow with its open end facing the jet at a distance H_4 below the exit at E. The dimensions H_1, H_2, H_3 and H_4 refer to physical dimensions of the apparatus while distances h_G and h_J refer to the liquid levels in the manometers.

Given that the liquid in the system is water, and that:
diameter of the the pipe from B to D, d_1 = 7 cm
diameter the second section from F to E, d_2 = 8 cm
h_A= 5 m
$H_2 = 3\,m$
$H_3 = H_4 = 1\,m$

1- Determine the flowrate Q in $\frac{m^3}{s}$

2- Determine the gage pressures p_C , p_D , p_F , and p_G at the respective points as shown in the figure.

3- Determine the the manometer readings h_G and h_J

4- Determine Q , h_G and h_J when the fluid in the system is a liquid with specific gravity SG = 0.7 instead of water, all other conditions remaining the same.

Solution

Q .1

Bernoulli between A and E

$$P_A + \rho \frac{V_A^2}{2} + \gamma z_A = P_E + \rho \frac{V_E^2}{2} + \gamma z_E$$

$$V_A = 0, \; P_A = p_E = p_{atm} = 0 \text{ gage}$$

$$V_E = \sqrt{2\,g\,h_A} = \sqrt{2 \times 9.81 \times 5} = 9.90 \, \frac{m}{s}$$

$$Q = V_E \frac{\pi}{4} d_2^2 = 9.90 \frac{\pi}{4} 0.08^2 = 0.0498 \, \frac{m^3}{s}$$

Q .2

By the equation of continuity $A_C\,V_C = A_E\,V_E$; $V_C = V_E(\,8/7)^2 = 9.9 \left(\frac{8}{7}\right)^2 = 12.93 \,\frac{m}{s}$ and $V_D = V_C$, and $V_F = V_G = V_E$

Bernoulli between A and different points, and deducing p (gage pressure)

$$\gamma\, z_A = P_C + \rho \frac{V_C^2}{2} + \gamma\, z_C;$$

$$p_C = \gamma\,(z_A - z_C) - \rho \frac{V_C^2}{2} = 9810\,(5) - \frac{1000}{2}\,(12.93)^2 = -34\,542 \text{ Pa}$$

$$\gamma\, z_A = P_D + \rho \frac{V_C^2}{2} + \gamma\, z_D;$$

$$p_D = \gamma\,(z_A - z_D) - \rho \frac{V_C^2}{2} = 9810\,(5-3) - \frac{1000}{2}\,(12.93)^2 = -63\,972 \text{ Pa}$$

But since $V_C = V_D$ we can also write directly

$$P_D = P_C + \gamma\,(z_C - z_D) = -34\,542 - 3 \times 9810 = -63\,972 \text{ Pa}$$

$$\gamma\, z_A = P_F + \rho \frac{V_E{}^2}{2} + \gamma\, z_F;$$

$$p_F = \gamma\,(z_A - z_F) - \rho \frac{V_E{}^2}{2} = 9810\,(5-3) - \frac{1000}{2}\,(9.9)^2 = -29\,385\ \text{Pa}$$

$$\gamma\, z_A = P_G + \rho \frac{V_E{}^2}{2} + \gamma\, z_G;$$

$$p_G = \gamma\,(z_A - z_G) - \rho \frac{V_E{}^2}{2} = 9810\,(5-1) - \frac{1000}{2}\,(9.9)^2 = -9765\ \text{Pa}$$

But since $V_F = V_G = V_E$ we can also write directly

$$P_F = 0 - 3 \times 9810 = -29\,430\ \text{Pa}$$

$$P_G = 0 - 9810 = -9810\ \text{Pa}$$

The differences in the answers is due to the rounding error in the value of the velocity

The pressures may also be expresed as absolute pressure by adding 101 kPa to the answers.

Q .3

$$\gamma\,(h_G - H_3) = p_G = \gamma\,(h_A - H_3) - \rho \frac{V_E{}^2}{2}\ ;\ h_G = h_A - \frac{V_E{}^2}{2\,g} = 5 - \frac{1}{2 \times 9.81}\,(9.9)^2 = 0$$

i.e. the level in the U tube to the right will be at the level 0 , same level than the exit E

The point at the entrance of the tube at J is the stagnation pressure, writing Bernoulli between E and J

$$p_J = \frac{\rho\, V_E{}^2}{2} + \gamma\,(z_E - Z_J) = \frac{\rho\, V_E{}^2}{2} + \gamma\, H_4 = \gamma\,\left(H_4 + h_j\right)$$

$$h_J = \frac{V_E{}^2}{2\,g} = \frac{1}{2\,g}\,(9.9)^2 = h_A = 5\ \text{m}$$

Q .4

Note that the expresssions for h_G may be written

$$h_G = h_A - \frac{V_E{}^2}{2\,g} = h_A - \frac{\sqrt{2\,g\,h_A}}{2\,g} = 0$$

This expression like the expression of h_J and for Q do not involve ρ or γ, and hence Q, h_G and h_J will remain the same irrespective of the fluid used.